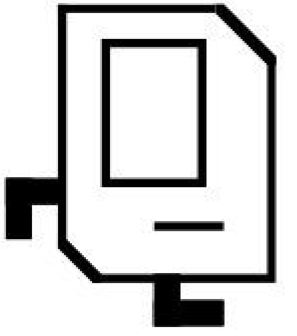
CS103
Fall 2025

Lecture 08:

Set Theory Revisited

Announcements!

CS198 Section Leading



cs198@cs.stanford.edu

Who should section lead?

For this round of applications, we are looking for applicants who have completed or are currently enrolled in CS106B.

We are looking for section leaders from all backgrounds who can relate to students and clearly explain concepts.

What do section leaders do?

- Teach a weekly 50 minute section
- Help students in the LaIR
- Grade CS106 assignments
- Hold IGs with students
- Grade midterms and finals
- Get paid $18.50/hour (more with seniority)
- Have fun!

Time and requirements

You'll need to:

- Section lead for **two quarters!**
- Take CS198 for 3-4 units (1st quarter only)
- Attend staff meetings (Monday, 4:30-5:30PM)
- Attend Monday workshops (7:30-9pm) for first 4 weeks of first quarter
- Attend Wednesday workshops (based on availability) for first 4 weeks of first quarter
- Fulfill all teaching, LaIR, and grading responsibilities

Why section lead?

- “Learn to teach; teach to learn”
- Work directly with students
- Participate in fun events
- Join an amazing group of people
- Leave your mark on campus

Participate in fun events

- LaIR Formal
- Special D
- BAWK
- Lecturer Hangouts
- New SL Picnic
- Swag
- And more!

Apply Now

Application is open now!

Deadlines:

Monday, October 13th at 11:59PM PT for students who have already completed CS106B or equivalent
Monday, October 27th at 11:59PM PT for current CS106B students

Online application: cs198.stanford.edu

Contact us: cs198@cs.stanford.edu

Thank You!

Problem Set Three

- Problem Set Two was due at 1:00PM today.
  - Need more time? You can use a late day to extend the deadline to 1:00PM Saturday.
- Problem Set Three goes out today. It’s due at 1:00PM next Friday.
  - Play around with functions and their properties.
  - Learn to write proofs on first-order definitions.
  - Explore the world of set theory in more detail.
- As usual, get in touch if you need help! Post on EdStem or stop by office hours. That’s what we’re here for.

Looking Forward: Midterm 1

- Our first midterm exam is a week from this Monday.
  - Details on Monday of next week.
- We've posted a big collection of practice exams on the course website (***Extra Practice Problems 1***).
- Students who need an alternate exam time: You should have already heard from us with your alternate time / location. If not, contact us ***ASAP*** (by the end of the day) to let us know.
- Students with OAE accommodations on exam: if we haven't received your OAE letter, send it to us ***ASAP*** (by the end of the day). OAE requires you to contact us at least ten days before a midterm exam.

On to CS103!

Outline for Today

- ***Proofs on Sets***
  - Making our intuitions rigorous.
- ***Formal Set Definitions***
  - What do our terms mean?
- ***Appendices: Examples***
  - Sample proofs to help you get the hang of the ideas here.

Recap from Last Time

	If you ***assume*** this is true...	To ***prove*** that this is true...
$\forall x.\ A$	Initially, ***do nothing***. Once you find a z through other means, you can state it has property A.	Have the reader pick an arbitrary x. We then prove A is true for that choice of x.
$\exists x.\ A$	Introduce a variable x into your proof that has property A.	Find an x where A is true. Then prove that A is true for that specific choice of x.
$A \to B$	Initially, ***do nothing***. Once you know A is true, you can conclude B is also true.	Assume A is true, then prove B is true.
$A \land B$	Assume A. Also assume B.	Prove A. Also prove B.
$A \lor B$	Consider two cases. Case 1: A is true. Case 2: B is true.	Either prove $\neg A \to B$ or prove $\neg B \to A$. *(Why does this work?)*
$A \leftrightarrow B$	Assume $A \to B$ and $B \to A$.	Prove $A \to B$ and $B \to A$.
$\neg A$	Simplify the negation, then consult this table on the result.	Simplify the negation, then consult this table on the result.

New Stuff!

Proving Results from Set Theory

Claim: If A, B, and C are sets where $A \in B$ and $B \in C$, then $A \in C$.

Proof (?): Assume A, B, and C are sets where $A \in B$ and $B \in C$. We need to show that $A \in C$.

Since $A \in B$, we know that A is contained in B. Since $B \in C$, we know that B is contained in C. Therefore, because A is contained in B and B is contained in C, we know that A is contained is C. This means that $A \in C$, as required. ■

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Which (if any) of these claims are true?
Answer at

https://cs103.stanford.edu/pollev

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This claim is not true! For example…

$$\emptyset \in \{\emptyset\}$$

$$\{\emptyset\} \in \{\{\emptyset\}\}$$

$$\emptyset \notin \{\{\emptyset\}\}$$

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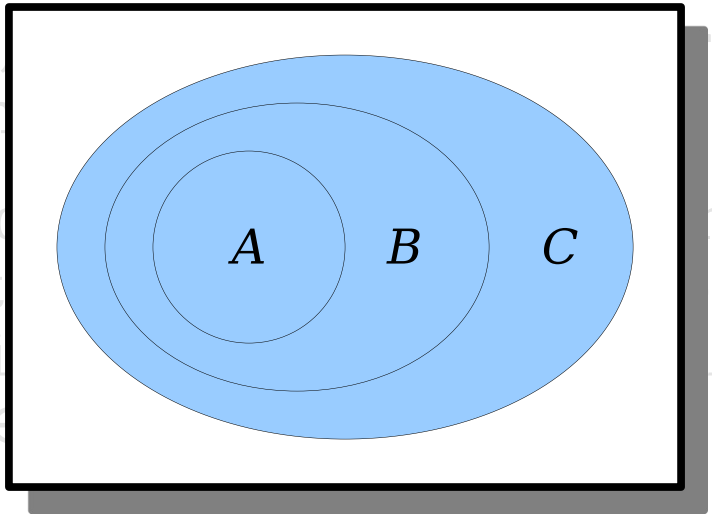

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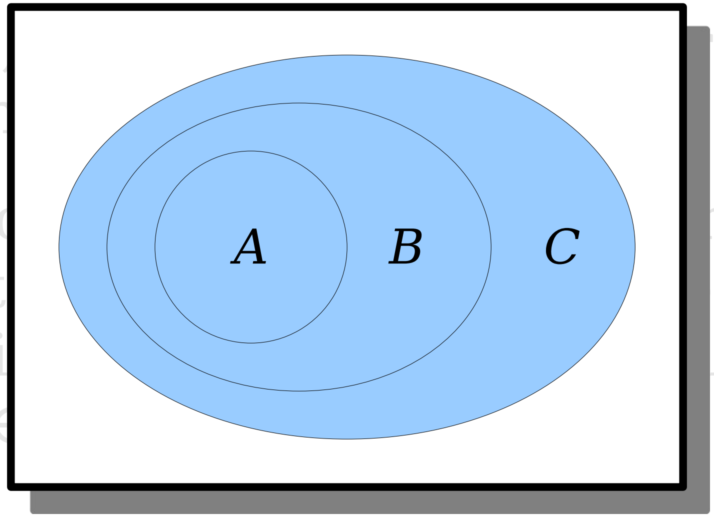

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This can't be a good proof; the same basic argument proves a false claim!

Claim: Let A, B, and C be sets. If $A \subseteq B$ and $A \subseteq C$, then we have $A \subseteq B \cap C$.

Proof (?): Assume $A \subseteq B$ and $A \subseteq C$. We need to show $A \subseteq B \cap C$.

Since $A \subseteq B$, all elements of A are in B. Since $A \subseteq C$, all elements of A are also in C. Therefore, all elements of A are in both B and C. Therefore, we see that $A \subseteq B \cap C$. ■

Claim: Let A, B, and C be sets. If $A \subsetneq B$ and $A \subsetneq C$, then we have $A \subsetneq B \cap C$.

Proof (?): Assume $A \subsetneq B$ and $A \subsetneq C$. We need to show $A \subsetneq B \cap C$.

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Which (if any) of these claims are true? Answer at

https://cs103.stanford.edu/pollev

This claim is not true! For example…

$$\{137\} \subsetneq \{137, 103\}$$
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But $\{137\} = \{137, 103\} \cap \{137, 107\}$

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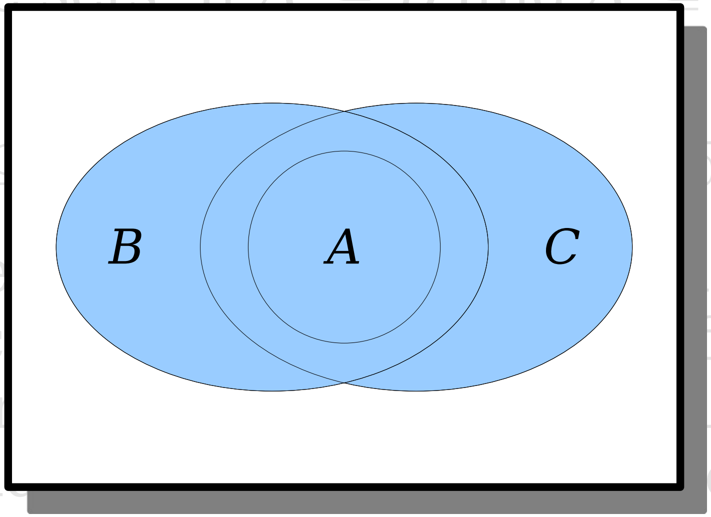

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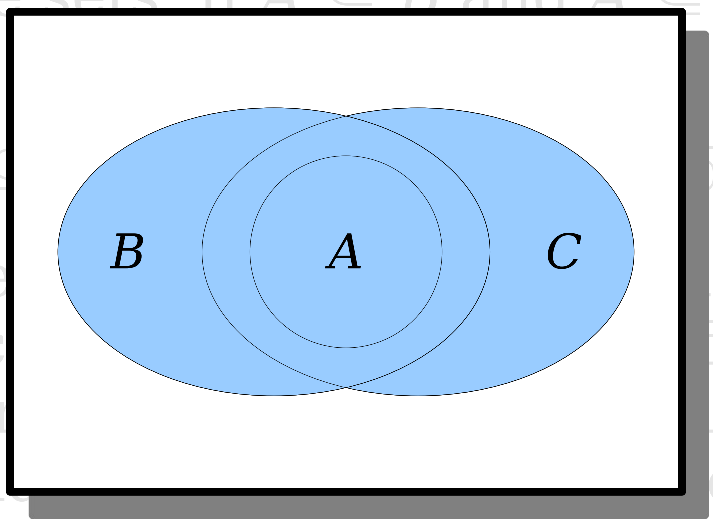

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What Went Wrong?

- The style of arguments you've just seen are ***not*** how to prove results on sets.
- As you've seen:
  - The reliance on high-level terms like "contained" is not mathematically precise.
  - A discussion of "all elements" of a set is not how to reason about collections of objects.
- ***Question:*** How do we write rigorous proofs about sets?

The Importance of Definitions

- As you’ve seen this week, formal definitions underpin mathematical proofs.
- The major issue from the previous proofs is that we haven’t defined what our terms mean.
  - How do we define what $A \in B$ means?
  - How do we define what $A \subseteq B$ means?
  - How do we define what $A \cap B$ means?
- Think back to our proof triangle: we currently have intuitions for these concepts, but not formal definitions.

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$			
$S = T$			
$x \in S \cap T$			
$x \in S \cup T$			
$X \in \wp(S)$			
$x \in \{ y \mid P(y) \}$			

Proofs on Subsets

Theorem: If A, B, and C are sets where $A \subseteq B$ and $B \subseteq C$, then $A \subseteq C$.

Defining Subsets

- Formally speaking, if S and T are sets, we say that $S \subseteq T$ when the following holds:

$$\forall x \in S.\ x \in T$$

- Now, suppose you're working with a proof where you encounter $S \subseteq T$. Think back to the proof table.
  - To ***assume*** that $S \subseteq T$, what should you do?
  - To ***prove*** that $S \subseteq T$, what should you do?

Answer at

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	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
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Since $x \in A$ and $A \subseteq B$, we know $x \in B$.

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A Correct Proof on Sets

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Unions and Intersections

Theorem: Let A, B, and C be sets. If $A \subseteq B$ and $A \subseteq C$, then $A \subseteq B \cap C$.

Unions and Intersections

- The statement $x \in S \cap T$ is defined as

$$\boldsymbol{x \in S \quad \wedge \quad x \in T.}$$

- The statement $x \in S \cup T$ is defined as

$$\boldsymbol{x \in S \quad \vee \quad x \in T.}$$

- These are operational definitions: they show how unions and intersections interact with the ∈ relation rather than saying what the union or intersection of two sets "are."

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$			
$x \in S \cap T$	$x \in S \ \wedge \ x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \ \vee \ x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
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Since $A \subseteq B$, all elements of A are in B. Since $A \subseteq C$, all elements of A are also in C. Therefore, all elements of A are in both B and C. Therefore, we see that $A \subseteq B \cap C$. ■

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Set Equality

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

	Is defined as...	If you ***assume*** this is true...	To ***prove*** that this is true...
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
$S = T$	$S \subseteq T \land T \subseteq S$	Assume $S \subseteq T$ and $T \subseteq S$.	Prove $S \subseteq T$. Also prove $T \subseteq S$.
$x \in S \cap T$	$x \in S \land x \in T$	Assume $x \in S$. Then assume $x \in T$.	Prove $x \in S$. Also prove $x \in T$.
$x \in S \cup T$	$x \in S \lor x \in T$	Consider two cases: Case 1: $x \in S$. Case 2: $x \in T$.	Either prove $x \in S$ or prove $x \in T$.
$X \in \wp(S)$			
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Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: See appendix!

Set-Builder Notation

Set-Builder Notation

- Let S be the set defined here:

$$S = \{ \ n \mid n \in \mathbb{N} \text{ and } n \geq 137 \ \}$$

- Now imagine you have some quantity x. Based on this…
  - … if you ***assume*** that $x \in S$, what does that tell you about x?
  - … if you need to ***prove*** that $x \in S$, what do you need to prove?

Answer at

https://cs103.stanford.edu/pollev

Set-Builder Notation

- Like unions and intersections, we have an operational definition for set-builder notation. It's the following:

Let $S = \{ y \mid P(y) \}$.
Then $x \in S$ when $P(x)$ is true.

- So, for example:
  - $x \in \{ n \mid n \in \mathbb{N} \text{ and } n \text{ is even} \}$ means $x \in \mathbb{N}$ and x is even.
  - $x \in \{ n \mid \exists k \in \mathbb{N}.\ n = 2k + 1 \}$ means that there is a $k \in \mathbb{N}$ where $x = 2k + 1$. (Equivalently, x is an odd natural number)
- ***Key Point:*** The placeholder variable disappears in all these examples. After all, *it's just a placeholder.*

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$X \in \wp(S)$			
$x \in \{ y \mid P(y) \}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Proofs on Set-Builder Notation

Some Useful Notation

- If n is a natural number, we define the set **[*n*]** as follows:

$$[n] = \{ \, k \mid k \in \mathbb{N} \ \land \ k < n \, \}$$

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- So, for example:
  - $[3] =$

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  - $[0] = \emptyset$
  - $[5] =$

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- So, for example:
  - $[3] = \{0, 1, 2\}$
  - $[0] = \emptyset$
  - $[5] = \{0, 1, 2, 3, 4\}$

Theorem: If $m, n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

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What We're Assuming	*What We Need to Prove*
$m \in \mathbb{N}$	$[m] \subseteq [n]$
$n \in \mathbb{N}$	
$m < n$	
$[z] = \{ k \mid k \in \mathbb{N} \land k < z \}$	

Theorem: If m, $n \in \mathbb{N}$ and $m < n$, then $[m] \subseteq [n]$

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$m \in \mathbb{N}$	~~$[m] \subseteq [n]$~~
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$x \in [m]$	

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$m \in \mathbb{N}$	~~$[m] \subseteq [n]$~~
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$m < n$	$x \in \mathbb{N}$
$[z] = \{ k \mid k \in \mathbb{N} \land k < z \}$	$x < n$
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Since $x \in [m]$, we know that $x \in \mathbb{N}$ and $x < m$. Then, because $x < m$ and $m < n$, we know that $x < n$.

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Notice that *there is no set-builder notation in this proof*. We were able to avoid it by using the rules for what $x \in \{ y \mid P(y) \}$ say to do.

	Is defined as…	If you ***assume*** this is true…	To ***prove*** that this is true…
$S \subseteq T$	$\forall x \in S.\ x \in T$	Initially, ***do nothing***. Once you find some $z \in S$, conclude $z \in T$.	Ask the reader to pick an $x \in S$. Then prove $x \in T$.
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$X \in \wp(S)$	$X \subseteq S$.	Assume $X \subseteq S$.	Prove $X \subseteq S$.
$x \in \{\, y \mid P(y) \,\}$	$P(x)$	Assume $P(x)$.	Prove $P(x)$.

Your Action Items

- ***Read "Guide to Proofs on Discrete Structures."***
  - There's additional guidance and practice on the assume/prove dichotomy and how it manifests in problem-solving.
- ***Read "Discrete Structures Proofwriting Checklist."***
  - Keep the items here in mind when writing proofs. We'll use this when grading your problem set.
- ***Read "Guide to Proofs on Sets."***
  - There's some good worked examples in there to supplement today's lecture, several of which will be relevant for the problem set.
- ***Start Problem Set 3.***
  - Start early and make slow and steady progress.
- ***(Optional) Look over Extra Practice Problems 1***
  - For extra pre-exam review.

Next Time

- ***Graph Theory***
  - A ubiquitous, powerful abstraction with applications throughout computer science.
- ***Vertex Covers***
  - Making sure tourists don’t get lost.
- ***Independent Sets***
  - Helping the recovery of the California Condor.

Appendix: More Sample Set Proofs

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming	*What I Need to Show*
$A \subseteq C$	$A \cup B \subseteq C \cup D$
$\forall z \in A.\ z \in C$	
$B \subseteq D$	
$\forall z \in B.\ z \in D$	

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming	*What I Need to Show*
$A \subseteq C$	~~$A \cup B \subseteq C \cup D$~~
$\forall z \in A.\ z \in C$	$\forall x \in A \cup B.\ x \in C \cup D$
$B \subseteq D$	
$\forall z \in B.\ z \in D$	

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming	*What I Need to Show*
$A \subseteq C$	~~$A \cup B \subseteq C \cup D$~~
$\forall z \in A.\ z \in C$	~~$\forall x \in A \cup B.$~~ $x \in C \cup D$
$B \subseteq D$	
$\forall z \in B.\ z \in D$	
$x \in A \cup B$	

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming	*What I Need to Show*
$A \subseteq C$	~~$A \cup B \subseteq C \cup D$~~
$\forall z \in A.\ z \in C$	~~$\forall x \in A \cup B.\ x \in C \cup D$~~
$B \subseteq D$	$x \in C$ or $x \in D$
$\forall z \in B.\ z \in D$	
$x \in A \cup B$	

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

What I'm Assuming	*What I Need to Show*
$A \subseteq C$	~~$A \cup B \subseteq C \cup D$~~
$\forall z \in A.\ z \in C$	~~$\forall x \in A \cup B.\ x \in C \cup D$~~
$B \subseteq D$	$x \in C$ or $x \in D$
$\forall z \in B.\ z \in D$	
$x \in A \cup B$	
Case 1: $x \in A$	
Case 2: $x \in B$	

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

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Proof:

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$.

Case 2: $x \in B$.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$. Then because $B \subseteq D$ and $x \in B$ we have $x \in D$, so $x \in C \cup D$.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$. Then because $B \subseteq D$ and $x \in B$ we have $x \in D$, so $x \in C \cup D$.

In either case, we see that $x \in C \cup D$, as required.

Theorem: Let A, B, C, and D be sets where $A \subseteq C$ and $B \subseteq D$. Then $A \cup B \subseteq C \cup D$.

Proof: Pick some $x \in A \cup B$; we need to show that $x \in C \cup D$.

Because $x \in A \cup B$, we know that $x \in A$ or $x \in B$. We consider each case separately.

Case 1: $x \in A$. Since $A \subseteq C$ and $x \in A$, we see that $x \in C$, and therefore that $x \in C \cup D$.

Case 2: $x \in B$. Then because $B \subseteq D$ and $x \in B$ we have $x \in D$, so $x \in C \cup D$.

In either case, we see that $x \in C \cup D$, as required. ■

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

(⇒) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$.

(⇐) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$.

Case 2: $x \in B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

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Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

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($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$. And since $x \in A \cup B$ and $A \cup B = B$, we see that $x \in B$, as required.

Theorem: Let A and B be sets. Then $A \subseteq B$ if and only if $A \cup B = B$.

Proof: We will prove each direction of implication.

($\Rightarrow$) Assume $A \subseteq B$. We need to show that $A \cup B = B$. To do so, we need to show that $A \cup B \subseteq B$ and that $B \subseteq A \cup B$.

First, we'll show $A \cup B \subseteq B$. Pick an $x \in A \cup B$. We need to show that $x \in B$. Since $x \in A \cup B$, we consider two cases:

Case 1: $x \in A$. Then since $x \in A$ and $A \subseteq B$, we have $x \in B$.

Case 2: $x \in B$. Then by assumption $x \in B$.

Either way, we have $x \in B$, which is what we needed to show.

Next, we'll prove $B \subseteq A \cup B$. Pick some $x \in B$. Since $x \in B$, we know that $x \in A \cup B$, as required.

($\Leftarrow$) Assume $A \cup B = B$. We need to show that $A \subseteq B$. So pick an $x \in A$; we need to show that $x \in B$.

Since $x \in A$, we know that $x \in A \cup B$. And since $x \in A \cup B$ and $A \cup B = B$, we see that $x \in B$, as required. ■

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming	*What I Need to Show*
$\wp(A) = \wp(B)$	$A = B$
$\wp(A) \subseteq \wp(B)$	
$\forall Z \in \wp(A).\ Z \in \wp(B)$	
$\wp(B) \subseteq \wp(A)$	
$\forall Z \in \wp(B).\ Z \in \wp(A)$	

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming	*What I Need to Show*
$\wp(A) = \wp(B)$	~~$A = B$~~
$\wp(A) \subseteq \wp(B)$	$A \subseteq B$
$\forall Z \in \wp(A).\ Z \in \wp(B)$	
$\wp(B) \subseteq \wp(A)$	$B \subseteq A$
$\forall Z \in \wp(B).\ Z \in \wp(A)$	

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming	*What I Need to Show*
$\wp(A) = \wp(B)$	~~$A = B$~~
$\wp(A) \subseteq \wp(B)$	~~$A \subseteq B$~~
$\forall Z \in \wp(A).\ Z \in \wp(B)$	$\forall x \in A.\ x \in B.$
$\wp(B) \subseteq \wp(A)$	~~$B \subseteq A$~~
$\forall Z \in \wp(B).\ Z \in \wp(A)$	$\forall z \in B.\ z \in A.$

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming	*What I Need to Show*
$\wp(A) = \wp(B)$	~~$A = B$~~
$\wp(A) \subseteq \wp(B)$	~~$A \subseteq B$~~
$\forall Z \in \wp(A).\ Z \in \wp(B)$	~~$\forall x \in A.$~~ $x \in B.$
$\wp(B) \subseteq \wp(A)$	~~$B \subseteq A$~~
$\forall Z \in \wp(B).\ Z \in \wp(A)$	$\forall z \in B.\ z \in A.$
$x \in A$	
$\{x\} \subseteq A$	
$\{x\} \in \wp(A)$	

Theorem: Let A and B be sets. Then if $\wp(A) = \wp(B)$, then $A = B$.

What I'm Assuming	*What I Need to Show*
$\wp(A) = \wp(B)$	~~$A = B$~~
$\wp(A) \subseteq \wp(B)$	~~$A \subseteq B$~~
$\forall Z \in \wp(A).\ Z \in \wp(B)$	~~$\forall x \in A.$~~ $x \in B.$
$\wp(B) \subseteq \wp(A)$	~~$B \subseteq A$~~
$\forall Z \in \wp(B).\ Z \in \wp(A)$	$\forall z \in B.\ z \in A.$
$x \in A$ $\quad$ $x \in B$	
$\{x\} \subseteq A$ $\quad$ $\{x\} \subseteq B$	
$\{x\} \in \wp(A)$ $\quad$ $\{x\} \in \wp(B)$	

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

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Proof:

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

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Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$. Thus we see that $\{x\} \subseteq B$, which in turn means that $x \in B$, as required.

Theorem: Let A and B be sets. If $\wp(A) = \wp(B)$, then $A = B$.

Proof: Assume $\wp(A) = \wp(B)$. We need to show that $A = B$, or, equivalently, that $A \subseteq B$ and $B \subseteq A$. Since the roles of A and B are symmetric, we'll just prove $A \subseteq B$.

Pick some $x \in A$; we need to show that $x \in B$. Since $x \in A$, we know that $\{x\} \subseteq A$. This means that $\{x\} \in \wp(A)$, and since $\wp(A) \subseteq \wp(B)$ we know $\{x\} \in \wp(B)$. Thus we see that $\{x\} \subseteq B$, which in turn means that $x \in B$, as required. ■